Modelling and Analysis of a Matrix-Reactance Frequency Converter Based on Buck-Boost Topology by DQ0 Transformation

Paweł Szczechniak*, Zbigniew Fedyczak* and Marius Klytta†
* University of Zielona Góra / Institute of Electrical Engineering, Zielona Góra, Poland, e-mail: P.Szcześniak@ieee.zu.zgora.pl, Z.Fedyczak@ieee.zu.zgora.pl.
† University of Applied Sciences / Faculty of Electrical Engineering, Giessen, Germany, e-mail: Marius.Klytta@ei.fh-giessen.de.

Abstract—This paper deals with a three-phase matrix-reactance frequency converter (MRFC). The analysed MRFC topology is based on buck-boost matrix-reactance chopper (MRC) one with source synchronous connected switches (LSCS) set arranged as in the step-up matrix converter (MC). The MRFC in question makes it possible to obtain a load output voltage much greater than the input voltage. Presented in this paper is a description a method for the analysis of the steady and transient state properties of presented MRFC. The static and dynamic characteristics of the presented converter under the control strategy proposed by Venturini are fully analysed on the basis of the circuit model development by the DQ0 transformation. Various static converter characteristics such as voltage and current gain, input power factor are completely analysed. Transition characteristics are also analysed by a small-signal model. The usefulness of the models is verified through computer simulations with good agreements.

Keywords—Matrix-reactance frequency converter, matrix converter, DQ0 transformation.

I. INTRODUCTION

One among the most desirable features of AC frequency converters is the generation of load voltage with arbitrary amplitude and frequency [1]. In recent years, matrix converters (MC) have received considerable attention as a competitor to the commonly used pulse width-modulated voltage-source inverter (PWM-VSI). One disadvantage of the MC is the voltage transfer ratio, which is limited to 0.5 of the input voltage [2] at linear voltage transformation, and to 0.866 or 1.053 at low-frequency load voltage deformations for space-vector or fictitious DC link control strategy concepts respectively [3], [4]. For application in the industrial drive field the maximum available magnitude of the output voltage should be even a little greater than the amplitude of the input voltage. In the case of the variable speed drive system for an induction motor, a reduction of the supply voltage by 10% means 20% loss of torque capability, which cannot be accepted in most applications. For application in FACTS a conventional auxiliary transformer employed to increase the MC output voltage is necessary. In reference [5] the concept of the step-up transformer employed to increase the MC output voltage is presented. In a circuit with this converter input inductors and output capacitors are used that function as input current sources and output voltage sources respectively. The voltage transfer ratio can be much greater than one, though voltage gain and input power factor cannot be controlled independently, they can be controlled by properly selected control parameters. Another concept of frequency converters based on buck-boost matrix-reactance chopper (MRC) is presented in references [6]-[9]. This concept is developed by authors and in reference [8] the generation concept of whole family of the matrix-reactance frequency converters based on unipolar PWM AC MRC is presented. The MRFC topologies are generated by use of the MRC one-cycle switched circuit models with suitable voltage and current sources introduced instead of the capacitors and inductors respectively. One of the synchronous connected switches (SCS) sets of the MRC is replaced by the step-down [2] or step-up [5] matrix-connected switches (MCS) set, offering the possibility of load voltage frequency change. A crucial fact is that all generated MRFCs have the possibility to obtain a load output voltage much greater than the input voltage. Actual problem deals with discussed MRFCs depend on researches of the properties these converters.

In this paper we obtained the analytic expressions for the voltage and current gain and input power factor of the MRFC based on buck-boost MRC, using DQ0 transformation technique [10]-[12]. By applying the DQ0 transformation, the three-phase balanced MRFC is transformed into a simple single-phase circuit model that does not contain a switch element. The analysis results are obtained for a classical Venturini control strategy. The main aim of this paper is description of the basic static and dynamic properties in order to an evaluation of the presented converter usefulness as an AC/AC frequency changer.

II. DESCRIPTION OF ANALYSED CONVERTER

A. MRFC Based on Buck-Boost MRC

The schematic diagram of the MRC with buck-boost topology is shown in Fig. 1 [9], [13] whereas one-cycle switched circuit models of this converter for two switches states are shown in Fig 2 [8]. In these models (Fig. 2) suitable voltage and current sources are taken into consideration instead of capacitors and inductors respectively. During the first switch state the source...
synchronous-connected switches (SSCS) are turn on and the load synchronous-connected switches (LSCS) are turned off. During the second switch-state the switches are in the inverse states.

![Schematic diagram of MRFC](image)

**Fig. 1.** Three-phase unipolar PWM AC MRC based on buck-boost topology

As is visible from Fig. 2, in each of the switch states, the SCS sets take part in two types of electrical energy transfer: from the voltage to the current sources; the second one as electrical energy is transferred from the current to the voltage sources. From an analysis of the electrical energy transfers it follows that one of SCS sets can be replaced by a MCS set, offering the possibility of the load voltage frequency change. The use of step-down or step-up of the MCS set is dependent on input and output voltage or current sources configurations [5].

During the first switches state electrical energy is transferred from the voltage to the current sources. In this case SSCS set can be replaced by a matrix connected switches (MCS) set with step-down configuration, offering the possibility of the load voltage frequency change. The schematic diagram of such MRFC (buck-boost I) is shown in Fig. 3 [7], [8]. The MCS output voltages $u_a$, $u_b$, and $u_c$ are formed according to (1).

$$
\begin{bmatrix}
u_a \\ u_b \\ u_c \\ \hat{u}_a \\ \hat{u}_b \\ \hat{u}_c \\ u_A \\ u_B \\ u_C \\ \hat{u}_A \\ \hat{u}_B \\ \hat{u}_C \\ u_{\text{A}} \\ u_{\text{B}} \\ u_{\text{C}} \\ \hat{u}_{\text{A}} \\ \hat{u}_{\text{B}} \\ \hat{u}_{\text{C}}
\end{bmatrix} = T
\begin{bmatrix}
s_{aB} \\ s_{aA} \\ s_{cA} \\ s_{bB} \\ s_{bA} \\ s_{cB} \\ s_{cC} \\ s_{aB} \\ s_{aA} \\ s_{cA} \\ s_{bB} \\ s_{bA} \\ s_{cB} \\ s_{cC} \\ u_{\text{A}} \\ u_{\text{B}} \\ u_{\text{C}} \\ \hat{u}_{\text{A}} \\ \hat{u}_{\text{B}} \\ \hat{u}_{\text{C}}
\end{bmatrix},
$$

where: $s_{jk} = \begin{cases} 1, & \text{switch } S_{jk} \text{ closed} \\ 0, & \text{switch } S_{jk} \text{ open} \end{cases}$, $j = \{a, b, c\}$, $K = \{A, B, C\}$, $T$ - instantaneous transfer matrix.

**Fig. 2.** One-cycle switched circuit models of three-phase unipolar PWM AC MRC based on buck-boost topology, a) for SSCS on and LSCS off, b) for SSCS off and LSCS on

During the second switches state electrical energy is transferred from the current to the voltage sources. Furthermore in both circuits the input low-pass filter $L_p$, $C_F$ is used in order to reduce the source current deformation.

During the second switches state electrical energy is transferred from the current to the voltage sources. In this case LSCS set can be replaced by a matrix connected switches (MCS) set with step-up configuration [5]. The concept of such MRFC (buck-boost II) is presented in [8].

**Fig. 3.** MRFC based on buck-boost MRC

A description of control strategy is general form is shown in Fig. 4a. The classical Venturini control strategy is taking into consideration with low frequency transfer matrix described by (2) [2]. Exemplary time waveforms of the control signals, illustrating operation of the discussed MRFC is shown in Fig. 4b.

**Fig. 4.** Control strategy: a) general form of control strategy description, b) exemplary time waveforms of the control signals for switches in one phase

$$
\mathbf{M} = \begin{bmatrix}
d_{aA} & d_{aB} & d_{aC} \\ d_{bA} & d_{bB} & d_{bC} \\ d_{cA} & d_{cB} & d_{cC}
\end{bmatrix},
$$

where:

$$
d_{aA} = d_{aB} = d_{aC} = D_s(1 + 2q \cos(\omega_s t + \varphi))/3,
$$

$$
d_{bB} = d_{bA} = d_{bC} = D_s(1 + 2q \cos(\omega_s t + \varphi - 2\pi/3))/3,
$$

$$
d_{cB} = d_{cA} = d_{cC} = D_s(1 + 2q \cos(\omega_s t + \varphi + 2\pi/3))/3.$$
\[d_{ac} = d_{ba} = d_{ab} = D_T(1 + 2q \cos(\omega_0 t + \varphi - 4\pi/3))/3,\]
\[d_{ck} = \text{low frequency component of the MCS switch state function},
D_T = t_s/T_{so} - \text{sequence pulse duty factor},
\omega_0 = \omega_0 - \omega, \\omega, \omega_1 = \text{pulsion of the supply and load}
\text{voltages respectively, } q = \text{voltage gain}.

III. MODELLING OF CONSIDERED TOPOLOGY

A. Averaged State Space Model

Assuming that all switches are ideal, inductors and capacitors are linear and
a so-called running average operator is used [14], then the mathematical
models of the proposed MRFC, for low frequency transfer matrix of MCS
according to classical Venturini control strategy expressed by (2), is described by
the matrix differential equation (3).

\[
\frac{d\mathbf{x}}{dt} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}(t),
\]

where:
\[
\mathbf{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_{r}(t) \\ x_{f}(t) \\ x_{b}(t) \end{bmatrix} - \text{vector of the averaged state variables,}
\mathbf{A}(t), \mathbf{B}(t) - \text{matrix and vector of MRFC parameters:}
\]

\[
\mathbf{A}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1/L_{r1} \\ 0 & 0 & 0 & 0 & 0 & 1/L_{r2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]
\[
\mathbf{B}(t) = \begin{bmatrix} u_{s1} \\ u_{s2} \\ u_{s3} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T
\]

It should be noted that equation (3), as a result of averaging, is continuously
non-stationary one.

From equations, (3) we can easily obtain the three-phase averaged circuit
model (Fig. 5). The equivalent circuit of switches would comprise ideal
transformers, whose turns ratios depend on the duty ratios.

B. The ABC-DQ0 Transformation

Sinusoidal time-varying systems can be changed to time-invariant system by
the DQ0 transformation [10], [11]. The DQ0 transformation of the variables is given as follows:

\[
x_{dq0} = \mathbf{K}x_{abc}, \quad x_{abc} = \mathbf{K}^{-1}x_{dq0},
\]

Fig. 5. Averaged circuit model of the considered MRFC

\[
\mathbf{K} = \frac{2}{\sqrt{3}} \begin{bmatrix} \cos(\omega_0 t) & \cos(\omega_0 t - 2\pi/3) & \cos(\omega_0 t + 2\pi/3) \\ \sin(\omega_0 t) & \sin(\omega_0 t - 2\pi/3) & \sin(\omega_0 t + 2\pi/3) \end{bmatrix},
\]

where: \(x_{dq0}, x_{r}, x_{f}, x_{b} \) - forward (rotating) phasor, \(x_{r} \) - backward (rotating) phasor, \(x_{f} \) - zero-sequence component.

As in the presented topology, there are two input and output work frequencies, we also have two transform matrices \(\mathbf{K} = \mathbf{K}_p \) and \(\mathbf{K}_l\) expressed by (6) [5], [12].

\[
\mathbf{K}_l = \frac{2}{\sqrt{3}} \begin{bmatrix} \cos(\omega_0 t) & \cos(\omega_0 t - 2\pi/3) & \cos(\omega_0 t + 2\pi/3) \\ \sin(\omega_0 t) & \sin(\omega_0 t - 2\pi/3) & \sin(\omega_0 t + 2\pi/3) \end{bmatrix}.
\]

The circuit DQ0 transformation is obtained by the following procedures:

- Partition of the averaged circuit model into basic subcircuits.
- Transformation of each of the subcircuits into DQ0 equivalent circuits based
  on the DQ0 transformation equations.
- Reconstruction of the transformed subcircuits by connecting the nodes of
  adjacent subcircuits.

C. Partition of the circuit into basic subcircuits

We can divide the averaged circuit model of the presented MRFC into several
fundamental subcircuits along the dotted lines indicated in Fig. 5. After partitioning,
we obtain eight basic subcircuits.

D. Transformation of Basic Subcircuits into DQ0 Equivalent Circuits

1) Circuit DQ0 Transform of Three-Phase Voltage
   Sources Set (Part A)

For a three-phase balanced voltage source set, the procedure is as follows:

\[
u_{dq0} = \mathbf{K}_p u_s = \mathbf{K}_p U_s \begin{bmatrix} \sin(\alpha + \varphi) \\ \sin(\alpha - 2\pi/3 + \varphi) \end{bmatrix} = U_s \begin{bmatrix} \sin \varphi \\ \sin(\alpha + 2\pi/3 + \varphi) \end{bmatrix}.
\]
inductor set (Part E): "inductor" is represented by real dynamic inductor and the circuit models are shown in Fig. 6b. The DQ0 formulated as:

\[ L_F \dot{i}_{Lpq0} = u_{Lpq0} \]

where \( L_{F1} = L_{F2} = L_{F3} = L_F \). Application of (4) and (5) to (8) yields:

\[ L_F \left( K_F^{-1} i_{Lpq0} + K_S^{-1} i_{Lpq0} \right) = u_{Lpq0} \]

Finally, the DQ0 transform of source inductors can be formulated as:

\[ L_F \dot{i}_{Lpq0} = -L_F K_S \left( K_S^{-1} i_{Lpq0} + K_S u_{Lpq0} \right) = -L_F \omega_0 \left( -1 0 0 \right) \]

and the circuit models are shown in Fig. 6b. The DQ0 "inductor" is represented by real dynamic inductor \( L_F \) in series with an imaginary static reactor \( ±j \omega_0 L_F \). Since the voltage and current of the static reactor obeys Ohm’s law, the reactor is replaced by a lossless resistor symbol [11].

Similar, equations and circuit models apply to the load inductor set (Part E):

\[ L_L \dot{i}_{Labc} = u_{Labc} \]

where \( L_{L1} = L_{L2} = L_{L3} = L_L \). From expression (4), (6) and (11) obtain:

\[ L_L \dot{i}_{Labc} = -L_L K_S \left( K_S^{-1} i_{Labc} + K_S u_{Labc} \right) \]

Figure 6c illustrates the DQ0 components of load inductors.

3) Circuit DQ0 Transform of Three-Phase Source and Load Capacitor Sets (Part C and Part G)

For the source capacitors circuit (Part C), the differential equations are in the following form:

\[ C_F \dot{u}_{Cpq0} = -C_F K_F \left( K_F^{-1} u_{Cpq0} + K_S i_{Cpq0} \right) \]

where \( C_{F1} = C_{F2} = C_{F3} = C_F \). Taking into account expressions (4), (5) and (13), the DQ0 transform of source capacitors is defined as follows:

\[ C_F \dot{u}_{Cpq0} = -C_F K_F \left( K_F^{-1} u_{Cpq0} + K_S i_{Cpq0} \right) \]

For the load capacitors circuit (Part G), the DQ0 transform is defined as follows:

\[ C_L \dot{u}_{Cpq0} = -C_L K_S \left( K_S^{-1} u_{Cpq0} + K_S i_{Cpq0} \right) \]

The DQ0 transformed circuit of source and load capacitor sets are shown in Fig. 6d and Fig. 6e, respectively. Similar as with inductors, the DQ0 capacitors are represented by real dynamic capacitors and \( C_L \) in parallel with imaginary static capacitors \( ±1/(j \omega C_L) \).

4) Circuit DQ0 Transform of Matrix Switches Set (Part D)

If the switching function of the matrix switches is defined by (2) then the DQ0 transformation of the nine switch matrix is given as follows [5], [12]:

\[ u_{sabc} = K_K \dot{u}_{sabc} = K_K M \dot{u}_{sabc} = K_K M \dot{u}_{sabc} = M \dot{u}_{sabc} \]

\[ M_{sabc} = K_K M_{sabc} = D_{sabc} = \]

The DQ0 transformed circuit of matrix switches set is shown in Fig. 6f.

5) Circuit DQ0 Transform of Load Switches Set (Part F)

If the switching function of the load switches is defined as:

\[ M_L = \]

then the DQ0 transform is described as follows (Fig. 6g):

\[ u_{Labc} = M_L \dot{u}_{Labc} \]

\[ K_L^{-1} \dot{u}_{Labc} = M_L K_L^{-1} \dot{u}_{Labc} \]

\[ u_{Labc} = M_L \dot{u}_{Labc} \]
6) Circuit DQ0 Transform of Three-Phase Load Resistor Set (Part H)

Assuming that, \( R_L = R_L = R_L \), the procedure of DQ0 transform of the resistor set is as follows (Fig. 6):

\[
\mathbf{u}_{Lq0} = \mathbf{K}_L \mathbf{u}_{abc} = \mathbf{K}_L \mathbf{R}_L \mathbf{i}_{abc} = R_L \mathbf{i}_{Lq0}
\]  

(23)

Fig. 6. DQ0 transformation of: a) voltage sources, b) source inductors, c) load inductors, d) source capacitors, e) load capacitors, f) matrix switches, g) load switches, h) load resistors

E. Circuit Reconstruction

The equivalent DQ0 circuit models of the presented MRFC based on buck-boost MRC (Fig. 3) are obtained as shown in Fig. 7 by rejoining of the DQ0 transformed subcircuits. Therefore, the three-phase circuit in Fig. 3 can be represented by three single-phase subcircuits for forward, backward and zero-sequence components.

Furthermore, assuming that the initial phase of input voltages equals zero \( \varphi = 0 \), and that the circuit is symmetrical and balanced, we obtain [6], [12]:

\[
\mathbf{u}_{Lq0} = \mathbf{K}_L \mathbf{u}_s = U_s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.
\]  

(24)

The equivalent circuits have been simplified from three circuits to one circuit, which is shown in Fig. 8.

IV. STEADY STATE ANALYSIS

The steady state model is obtained simply by eliminating the reactive elements. With reference to Fig. 9, the inductors seem to be short and capacitors open. The steady state characteristics can be obtained by considering the circuit model of the presented MRFC. For steady state analysis a single-phase circuit model is divided into four terminal networks (Fig. 9) [9], [15]. With reference to Fig. 9 four-terminal chain equations in complex form can be written as (25):

\[
\Delta_{LF} = \begin{bmatrix} 1 & j \omega L_F \\ 0 & 1 \end{bmatrix}, \quad \Delta_{LS} = \begin{bmatrix} 1 & j \omega L_S \\ 0 & 1 \end{bmatrix}
\]

\[
\Delta_{CF} = \begin{bmatrix} 1 & 0 \\ j \omega C_F & 1 \end{bmatrix}, \quad \Delta_{CL} = \begin{bmatrix} 1 & 0 \\ j \omega_C & 1 \end{bmatrix}
\]

\[
\Delta_{TRI} = \begin{bmatrix} 1 & 0 \\ qD_s & 0 \end{bmatrix}, \quad \Delta_{TR2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

(25)

Fig. 7. DQ0 transformation of three phase MRFC based on buck-boost MRC (Fig. 1): a) forward sequence component, b) backward sequence component, c) zero-sequence component

Fig. 8. DQ0 transformation of three phase MRFC based on buck-boost MRC (Fig. 1) for \( \varphi = 0 \), and balanced-symmetrical circuit condition

Fig. 9. Steady state equivalent circuit for MRFC based on buck-boost MRC: \( \Delta_{Ls}, \Delta_{Ls} \) chain matrix for the source and load inductors respectively, \( \Delta_{TR1}, \Delta_{TR2} \) chain matrix for source and load transformer respectively, \( \Delta_{TFS}, \Delta_{TLC} \) chain matrix for the source and load capacitors respectively
Applying the four-terminal network description method [9], [15] we obtain:

\[
\begin{bmatrix}
U_s \\
L_s
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
U_i \\
L_i
\end{bmatrix},
\]

(26)

\[
A = A_{11} \Delta \omega \Delta_{rb} A_{12} \Delta_{rb} \Delta_{cl},
\]

(27)

where:

\[
A_{11} = \frac{j \omega \Delta L_s + \omega C_L \left(1 - j \omega \Delta C_L, L_s \right)}{q(1 - D_s)D_s},
\]

\[
A_{12} = \frac{j \omega \Delta C_L \Delta L_s + \omega C_L \left(1 - j \omega \Delta C_L, L_s \right)}{q(1 - D_s)D_s},
\]

\[
A_{21} = \frac{j \omega \Delta C_L \Delta L_s + \omega C_L \left(1 - j \omega \Delta C_L, L_s \right)}{q(1 - D_s)D_s},
\]

\[
A_{22} = \frac{j \omega \Delta C_L \Delta L_s + \omega C_L \left(1 - j \omega \Delta C_L, L_s \right)}{q(1 - D_s)D_s}.
\]

In accord with four-terminal theory we obtain equations (28)-(30) [15]:

\[
|H_V| = \frac{U_s}{U_i} = \left| \frac{1}{A_{11} + A_{12}} \left( R_L \right) \right|,
\]

(28)

\[
|H_I| = \frac{L_s}{L_i} = \left| \frac{1}{A_{21} + A_{22}} \left( R_L \right) \right|,
\]

(29)

\[
\lambda_p = \frac{P_s}{S_s} = \cos \left[ \arg \left( \frac{A_{11}Z_L + A_{12}}{A_{21}Z_L + A_{22}} \right) \right].
\]

(30)

The characteristics of magnitude of voltage and current transmittance and input power factor as functions of load voltage setting frequency and pulse duty factor \(D_s\), obtained by means of (25)-(30) for circuit parameters collected in the Appendix A in Table 1, are shown in Figs. 10-12, respectively. For the purpose of comparison these characteristics are presented together with ones obtained by means of simulation investigations of the presented circuit with idealized switches (Fig. 3). The simulation investigations have been carried out with the help of the program PSpice. As is visible from Figs. 10-12 properties of the MRFC are strongly dependent on parameters of the passive elements of the discussed circuit. Generally the results of the simulation investigation confirm the results of theoretical studies. Differences between analytic and simulation results are caused by higher harmonics being taken into account during the simulation investigation (non-stationary circuit).

The presented results have been obtained in near matching conditions described by (31) [9].

\[
R_L = 3 \sqrt{L_{10}/C_{10}},
\]

(31)

\[
\lambda_p = \frac{P_s}{S_s} = \cos \left[ \arg \left( \frac{A_{11}Z_L + A_{12}}{A_{21}Z_L + A_{22}} \right) \right].
\]

(30)

V. TRANSIENT-STATE ANALYSIS

The equivalent circuit of transient-state is shown in Fig. 8. For this circuit we obtain the steady-state averaged state-space equation (32) [14].

\[
\begin{bmatrix}
\frac{d \bar{i}_1}{dt} \\
\frac{d \bar{i}_{12}}{dt} \\
\frac{d \bar{i}_{21}}{dt} \\
\frac{d \bar{i}_{22}}{dt} \\
\frac{d \bar{i}_{23}}{dt}
\end{bmatrix} =
\begin{bmatrix}
-j \omega & 0 & -\frac{1}{L_s} & 0 \\
0 & -j \omega & \frac{qD_s}{C_L} & \frac{(1 - D_s)}{L_s} \\
0 & \frac{qD_s}{C_L} & -j \omega & 0 \\
0 & 0 & -\frac{1}{R_C} & -j \omega \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\bar{i}_1 \\
\bar{i}_{12} \\
\bar{i}_{21} \\
\bar{i}_{22} \\
\bar{i}_{23}
\end{bmatrix} +
\begin{bmatrix}
\frac{1}{R_L} \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\bar{u}_1 \\
\bar{u}_{12} \\
\bar{u}_{21} \\
\bar{u}_{22} \\
\bar{u}_{23}
\end{bmatrix}.
\]

(32)
Assuming that all variables have two components: a running constant component (the averaged value in the switching period $T_{Seq}$), which is marked by upper case letter, and a perturbation one marked by lower case letter, which is covered by the sign “$\hat{}$”:

$$u = U + \hat{u}, \quad x = X + \hat{x}, \quad d = D + \hat{d} \quad (33)$$

The small signal state space equations are expressed as follows [14]:

$$\frac{d}{dt}(X + \hat{x}) = A\hat{x} + B\hat{u} + [(A_1 - A_2)X + (B)U]d \quad (34)$$

where $A_1 = A(D_S = 1), A_2 = A(D_S = 0)$.

According to (34) Laplace transform of a small signal state-space equation is expressed as (35).

$$\tilde{s}\hat{x}(s) = A\hat{x}(s) + B\hat{u}(s) + [(A_1 - A_2)X + (B)U]\hat{d}(s) \quad (35)$$

After rearrangement there is:

$$\tilde{x}(s) = (sI - A)^{-1}[B\hat{u}(s) + [(A_1 - A_2)X + (B)U]\hat{d}(s)] = \mathbf{G}_{\tilde{x}\tilde{u}(s)}(s)\hat{u}(s) + \mathbf{G}_{\tilde{x}\tilde{d}(s)}(s)\hat{d}(s) \quad (36)$$

We obtain two following perturbation transform functions:

$$\mathbf{G}_{\tilde{x}\tilde{u}(s)}(s) = \tilde{x}(s)\tilde{u}(s), \quad (37) \quad \mathbf{G}_{\tilde{x}\tilde{d}(s)}(s) = \frac{\tilde{x}(s)}{\tilde{d}(s)} \quad (38)$$

Detailed definitions of equations (37) and (38) are presented in Appendix B.

Presented in Fig. 13 are the transient responses of state variables at a step change of the load frequency from 25Hz to 50Hz, for summarized pulse duty factor equal $D_S = 0.75$.

Fig. 13. Transient responses of states variables at step change of the output frequency $f_L$ from 25Hz to 50Hz, for $D_S = 0.75$

Represented in Fig. 14 are the calculation and simulation test results of transient responses of state variables for two different output frequencies, 25 and 75Hz. Presented in both cases is the step change of the input voltages from 50% to 100% of their nominal values in time moment $t_0$ and pulse duty factor equal $D_S = 0.75$.

Whereas, in Fig. 15 the transient responses of state variables at a step change of the sequence pulse duty factor $D_S$ from 0.5 to 0.75, for $f_L = 25$Hz are presented.

Figures 13, 14 and 15 show good consistency of calculation and simulation test results. The obtained results confirm that small signal models can be useful for transient response analysis of the described MRFC.

Fig. 14. Transient responses of states variables at step change of the supply voltage at $D_S = 0.75$ a) for output frequency 25Hz b) for output frequency 75Hz

Fig. 15. Transient responses of states variables at step change of the sequence pulse duty factor $D_S$ from 0.5 to 0.75, for $f_L = 25$Hz
VI. CONCLUSIONS

The steadystate and small signal mathematical and circuit models of MRFC with buck-boost topology have been elaborated. Furthermore, the steady state characteristics and transient responses of the analysed circuit have also been investigated. Simulation test results, obtained for MRFC with idealized switches, have confirmed that elaborated models can be useful with respect to steady state and transient responses of MRFC topology. The validity of the proposed models will be the subject of future investigations of the presented MRFC with active load and for a closed control system too.

APPENDIX A

TABLE I. THEORETICAL AND SIMULATION TESTS CIRCUIT PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply voltage/frequency</td>
<td>$U_{dc}$</td>
<td>230 V/50 Hz</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>$f_s$</td>
<td>5 kHz</td>
</tr>
<tr>
<td>Inductances</td>
<td>$L_{r1}$</td>
<td>0.5 mH</td>
</tr>
<tr>
<td>Capacitances</td>
<td>$C_{D1}$</td>
<td>50 μF</td>
</tr>
<tr>
<td>Load resistance</td>
<td>$R_L$</td>
<td>10 Ω</td>
</tr>
</tbody>
</table>

REFERENCES


APPENDIX B

SMALL SIGNAL TRANSFER FUNCTIONS OF ANALYZED MRFC

\[
G_{\omega} = \frac{D_{2} G_{1}^{+} \left[ 1 + R_{C1} C_{L} \left( s + j \omega \right) \right]}{R_{C1} C_{L} \left( s + j \omega \right) \det(s - A)}
\]

\[
G_{\omega} = \frac{D_{2} G_{1}^{+} \left[ 1 + R_{C1} C_{L} \left( s + j \omega \right) \right]}{G_{1} + G_{L} \left( s + j \omega \right)}
\]

where: $D_{1} = \left( D_{2} - 1 \right)$, $X_{L} = j \omega L_1$, $G_{i} = a \frac{D_{2} G_{1}^{+} \left[ 1 + R_{C1} C_{L} \left( s + j \omega \right) \right]}{R_{C1} C_{L} \left( s + j \omega \right) \det(s - A)}$, $D_{2} G_{1}^{+} \left[ 1 + R_{C1} C_{L} \left( s + j \omega \right) \right] = G_{1} + G_{L} \left( s + j \omega \right)$.